

Midterm 2 – 60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

1. (20 Points) Suppose that you wish to predict wage outcomes via the following specification:

$$wage = \beta_0 + \beta_{educ} educ + u$$

wage is measured in dollars per month, and *educ* is measured in years

a.) What does OLS stand for? (2 Points)

b.) To generate estimates, you solve $\min_{\beta_0, \beta_{educ}} \sum_{i=1}^n \hat{u}_i^2$, calculating values of $\hat{\beta}_0$ and $\hat{\beta}_{educ}$. What, if anything, is wrong with your procedure? (4 points)

c.) After estimating by OLS, the predicted value of *wage* is written as: (4 Points)

$$E[wage | educ] = \hat{\beta}_0 + \hat{\beta}_{educ} educ$$

What happened to *u* and why?

d.) Suppose you estimate $\hat{\beta}_{educ} = 100$ and $\hat{\sigma}_{wage,educ} = 400$. What is the standard deviation of *educ*? (10 Points)

2. (20 Points) Using a slightly different model as (1), you now wish to estimate:

$$\log(\text{wage}) = \beta_0 + \beta_{educ} \log(\text{educ}) + u$$

a.) Suppose you estimate that $\hat{\beta}_{educ} = 3$. Please interpret this estimate. (5 Points)

b.) Please prove (by taking derivatives) that changing the units of *wage* from dollars to thousands of dollars will not affect $\hat{\beta}_{educ}$. (10 Points)

c.) Suppose that the variance of *u* changes systematically with things like *hair color*, but does not change systematically with *educ*. What type of errors are these? (5 Points)

3. (20 Points) In an effort to predict how you will do in 113, you ask Professor Spearot to provide evidence about how other students did in his class as a function of prior performance in school, and the first midterm score. He plans to estimate the following equation by OLS

$$Grade = \beta_0 + \beta_{MT1}MT1 + \beta_{GPA}GPA + u$$

Here, *Grade* is the final grade in the course (0-100), *MT1* represents the percentage point score on midterm 1 (0-100), and *GPA* represents student GPA (between 0 and 4) prior to enrolling in 113. Professor Spearot reports that $\hat{\beta}_{MT1} > 0$ and $\hat{\beta}_{GPA} > 0$.

a. Professor Spearot forgot to record previous experience in math/statistics before collecting data at the registrar's office. The variable *MATH* measures the number of classes a student has taken in math or applied math fields. Supposing that prior experience (*MATH*) is positive correlated with *MT1* and *Grade*, what is the direction of the bias in $\hat{\beta}_{MT1}$? Based on this information, can I be confident that $\beta_{MT1} > 0$? (5 Points)

b. Further, suppose that Math professors tend to give lower grades on average (harder material, cranky professors). Along with any relevant information in (a), supposing that prior experience (*MATH*) is negatively correlated with GPA, what is the direction of the bias in $\hat{\beta}_{GPA}$? Based on this information, can I be confident that $\beta_{GPA} > 0$? (5 Points)

For the remainder of this question, assume that $\hat{\beta}_0 = 20$, $\hat{\beta}_{MT1} = 0.4$, and $\hat{\beta}_{GPA} = 10$

- c. What is the predicted final grade for somebody with a 3.4 GPA and an 80 on the first midterm? Is this a sensible prediction?
- d. You need to get a 90 or above for a final grade to be accepted to a graduate program in dismal sciences. Your pre-113 GPA is 3.2. What is the minimum MT1 score such that your expected final grade is at or above 90?

Please enjoy your weekend.